MATHEMATICAL MODELING OF HEAT AND MASS TRANSFER IN TRANSPIRATION COOLING OF WATER DROPLETS IN A COOLING TOWER

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Mathematical modeling of the processes of heat and mass transfer in transpiration cooling of water droplets is carried out. In a self-consistent approximation the variation of the temperature and density of the water vapor in the vapor-air mixture of a flowing droplet as it falls in the water distributing space of a cooling tower is taken into account.

Introduction. Interest in various aspects of operation of cooling towers, growing in recent years [1-4], is largely related to problems of power resource saving in power engineering and the effort to increase the efficiency of cooling towers [5-7]. Complex interdependent processes of motion and heat and mass transfer between warm water and a vapor-air mixture render a direct approach to investigating the aerodynamics and heat and mass transfer of a cooling tower impossible. Therefore both laboratory modeling [2, 8] and simplified mathematical models are growing in importance.

The present work considers the motion and heat and mass transfer of an ensemble of droplets of the same dimension, falling toward an upward vapor−air flow. The coefficients of heat transfer α and mass transfer γ of a single droplet depend, in this case, on the Reynolds number Re according to the law [9]:

$$\alpha \sim \mathrm{Re}^{1/2},\ \ \gamma \sim \mathrm{Re}^{1/2}.$$

Mathematical Model. We introduce the axis of ordinates z in such a way that the free fall acceleration $g = -9.8$ m/sec^2, and the origin of coordinates coincides with the surface of a drainage basin. Then the system of equations describing interrelated the processes of heat and mass transfer and motion has the form

$$\frac{dx}{dt} = v; \tag{1}$$

$$\frac{d(mv)}{dt} = -mg + C_D(\mathrm{Re})\frac{\rho_a (u-v)^2}{2}\pi R^2; \tag{2}$$

$$\frac{d(mcT)}{dt} = 4\pi R^2[\alpha(\mathrm{Re})(T - T_a(z=x)) + r\gamma(\mathrm{Re})(\rho(z=x) - \rho_s(T(x)))]; \tag{3}$$

$$\frac{dm}{dt} = 4\pi R^2\gamma(\mathrm{Re})(\rho(z=x) - \rho_s(T(x))). \tag{4}$$

The initial conditions for the system of equations (1)-(4) have the form for $t = 0$

$$x = H,\ v = v_0,\ T = T_0,\ m = \frac{4\pi}{3}\rho_L R_0^3. \tag{5}$$

We note that $T_a(z = x)$ is the temperature of the vapor−air mixture in the vicinity of the droplet with the center coordinate x. The specific flow rate of water Q in the cooling tower is then equal to

$$Q = N_0 v_0 m,$$

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where N_0 is the initial number of droplets in a unit volume. By virtue of the retained number of droplets we have

$$N_0 v_0 = N(x) v(x).$$

Thus, as the velocity of the droplets increases, their density N(x) decreases. In most modern cooling towers the droplet density in the volume unit is such that

$$RN^{1/3}(x) \ll 1,$$

and, consequently, applicability of the approximation of a single droplet is beyond question. Let us turn to the description of the procedure of self-consistency. We denote by $Q^{\omega}(z)$ the specific flow rate of water vapor in the cooling tower cross section at the height z and by $Q_t(z)$ the specific thermal energy flux. Assuming the velocity of the vapor–air stream within the cooling tower u to be constant and considering it hereinafter as a parameter, $Q_{\omega}(z)$ and $Q_T(z)$ can be represented in the form:

$$Q_w(z) = u\rho(z), \quad Q_T(z) = u\rho_a c_a T(z).$$

We assume linear profiles of temperature and density of water vapor in the water distributing space of the cooling tower:

$$T(z) = T_a + \frac{T(H) - T_a}{H} z; \tag{6}$$

$$\rho(z) = \rho_s(T_a)\varphi + \frac{\rho(H) - \rho_s(T_a)\varphi}{H} z. \tag{7}$$

By solving the system of equations (1)-(5) with the temperature and density profiles (6) and (7) one can find values of T(H) and ρ(H) such that

$$Q_w(H) - Q_w(0) = \frac{4\pi}{3}\rho_L N_0 v_0 (R_0^3 - R_f^3), \tag{8}$$

$$Q_T(H) - Q_T(0) = \frac{4\pi}{3}\rho_L c N_0 v_0 (R_0^3 T_0 - R_k^3 T_f), \tag{9}$$

where R_f and T_f are the solution of the system of equations (1)-(5) for x = 0. Relations (8) and (9) have a simple physical sense: they denote that the overall thermal energy and water vapor, released by the cooling of the droplets at the distance H, are carried by the upward vapor–air flow. In this case, we make the insignificant assumption that the water density ρL and the heat capacity c are constant in the temperature range in question.

As is well known [9], for a droplet the Nusselt number depends on the Reynolds number according to the law Nu = 2 + 0.5Re$^{1/2}$, where Nu = 2Rα(Re)/λ, and the Sherwood number Sh = 2 + 0.5Re$^{1/2}$, where Sh = 2Rγ(Re)/D. The Reynolds number Re = ρ_a2R|(u − v)|/μ. Here λ and μ are, respectively, the thermal conductivity and viscosity of air. Thus, when Re > 10^2, $\gamma \sim DR^{-1/2}(v - u)^{1/2}$. The coefficient of resistance of the droplet C_D was calculated by the formula:

$$C_D = \frac{16}{\text{Re}}\left(1 + \frac{1}{6}\text{Re}^{2/3}\right).$$

Results and Discussion. Before passing to the presentation of the results of numerical calculations, we perform a qualitative investigation of the system of equations (1)-(5). It can easily be shown that in a time of the order of 1 sec droplets with radii larger than 0.001 m gather a steady-state velocity $v_s(R)$ such that the right side of Eq. (2) becomes zero, and the droplets move further with a constant velocity equal to the steady-state one. Then, approximately integrating, in view of this fact and Eqs. (3) and (4) and taking into account solely the transpiration cooling, it is easy to obtain the following relations:

$$T_f - T_0 = \Delta T \sim \frac{3r\gamma(\text{Re}_y)\rho_s(T_0)}{R_0\rho_L c}\left[\frac{1}{S} - 1\right]\frac{H}{v_s} \equiv \Delta T_0; \tag{10}$$

$$R_f - R_0 = \Delta R \sim \frac{\gamma(\text{Re}_y)\rho_s(T_0)}{\rho_L}\left[\frac{1}{S} - 1\right]\frac{H}{v_s}. \tag{11}$$

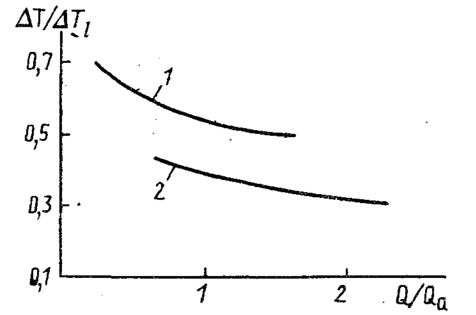

Fig. 1

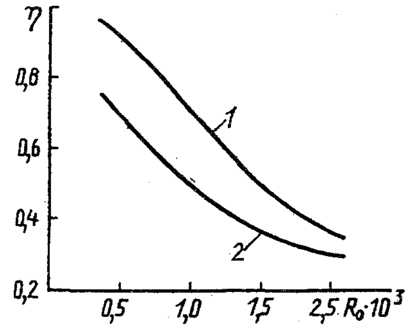

Fig. 2

Fig. 1. Dependence of the cooling tower efficiency $\eta = \Delta T/\Delta T_l$ on the ratio between the specific flow rates of water and air (the result of calculation for N = 16 m, S = 3.9 (φ = 0.7): 1) $R_0 = 10^{-3}$ m; 2) $1.5 \cdot 10^{-3}$.

Fig. 2. Cooling tower efficiency η vs initial droplet radius R_0 (the result of the calculations for H = 16 m, S = 3.9; Q/Q_a = 1): 1) zero approximation, 2) self-consistent approximation. R, m.

In expressions (10) and (11), from our viewpoint, an important notion has emerged – the degree of nonequilibrium S [10]:

$$S = \frac{\rho_s(T_0)}{\rho_s(T_a)\,\varphi}. \tag{12}$$

The more S deviates from unity, the more intense are the processes of transpiration cooling. We also note a fairly strong dependence of ΔT, as follows from (10), on the droplet radius R_0. It is common knowledge that an important parameter in the theory of transpiration cooling [5] is the limiting temperature T_l, determined from the condition

$$\rho_s(T_l) = \varphi\rho_s(T_a). \tag{13}$$

It is convenient to characterize the performance of the cooling tower by the quantity η, which is a peculiar kind of efficiency:

$$\eta = \frac{\Delta T}{\Delta T_l} = \frac{T_f - T_0}{T_l - T_0} < 1. \tag{14}$$

We come to the presentation of the results of numerical calculations for the system (1)-(5) with the procedure of self-consistency (8), (9). The system of equations (1)-(5) was initially solved in the zero approximation: the temperature of the vapor–air mixture and the density of water vapor in the water distribution space of the cooling tower were taken equal to the air temperature and the density of the water vapor near the cooling tower. Then using an iteration procedure and expressions (6)-(9) the self-consistent solution of the starting system of equations was found. Usually 7-8 iterations were required. It is natural that in the zero approximation the value of η is maximal; hence the zero approximation is of interest for estimating the maximal operating efficiency of the cooling tower.

Figure 1 shows the dependence of the cooling tower operating efficiency, calculated in the self-consistent approximation, on the ratio between the specific flow rates of water Q and air $Q_a = \rho_a u$. It is reasonable that with increasing water concentration the performance of the cooling tower monotonically drops, while its value strongly depends on R_0 and H (see (10)). On the other hand, as the specific flow rate of the air increases, the performance of the cooling tower increases at the expense of increasing both the fall time of the droplets and the evaporation rate.

Figure 2 gives the results of the calculation of the cooling tower efficiency for $Q/Q_a = 1$ in the zero and self-consistent approximation versus the droplet radius. It is notable that for droplets with $R_0 \geq 2 \cdot 10^{-3}$ the difference in efficiency between the zero and self-consistent approximation is less than 7%. This is related to the fact that fairly large droplets fall rapidly and, with the constraint Q/Q_a = const, little disturb the state of the vapor–air mixture entering the cooling tower. As a result, large droplets are poorly cooled in comparison with smaller ones.

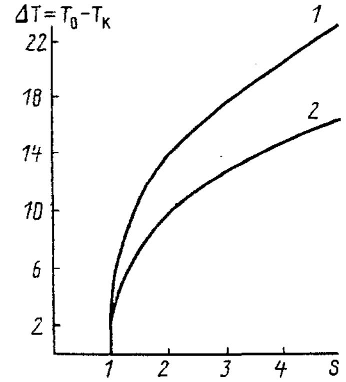

Fig. 3. Dependence of the temperature drop ΔT of water attained in the cooling tower on the degree of nonequilibrium S (result of the calculations for H = 16 m; $R_0 = 10^{-3}$ m, $Q/Q_a = 1$): 1) zero approximation, 2) self-consistent approximation.

Figure 3 gives the calculations of the dependence of the modulus of ΔT on the degree of nonequilibrium S, which takes into account the initial water temperature and the temperature and relative humidity of the air surrounding the cooling tower. We note that the higher the degree of nonequilibrium, the larger the discrepancy between the zero and self-consistent approximations. Generalizing the results of [3, 5], it is important to point out that, as the results of the calculations show, the value of η remains practically unchanged when varying the degree of nonequilibrium S over wide limits with $S \neq 1$.

As comparison of expression (10) with the results of numerical calculations shows, $\Delta T/\Delta T_0 \sim (0.7\text{-}0.8)$ if $R_0 > 1.5 \cdot 10^{-3}$ m and $Q/Q_a \leq 2$. The smaller the latter ratio, the more accurately ΔT_0 describes the temperature variation of the droplets.

The application of other (nonlinear) profiles of temperature and density of vapor in combination with the conditions (8) and (9) has shown that the results on ΔT are found between the zero approximation and the self-consistent one with a linear profile.

It is interesting to note that from the results of the calculations the ratio of the specific flow rate of the evaporated liquid Q_e to the specific flow rate of the water Q is equal to:

$$Q_e/Q \sim (0{,}01 - 0{,}02),$$

which is in good agreement with the approximate equality following from the law of energy conservation:

$$Q_e/Q \simeq \frac{c\Delta T}{r},$$

and, as is evident from (8) and (11), strongly depends on R_0, the degree of nonequilibrium S, the initial water temperature, and H.

We note a further important, though indirect, conclusion from our calculations. Heating of the vapor-air mixture is done mainly by recondensation of hot vapor in mixing with the relatively cold vapor-air mixture entering from the environment.

CONCLUSIONS

1. By the calculations within the framework of the formulated mathematical model of heat and mass transfer in a cooling tower it is shown that the thermal efficiency of the cooling tower η in a stationary regime decreases monotonically as the ratio of the specific flow rates of the water and the air Q/Q_0 increases. As $Q/Q_a \to 0$, in its turn $\eta \to 1$.

2. The value of the temperature drop ΔT of water, resulting from cooling, depends in a nonlinear manner on the initial temperature of water for small S:

$$\Delta T \sim \rho_s(T_0)\left(\frac{1}{S}-1\right)HD.$$

3. The value of the temperature drop of droplets is substantially affected by the droplet radius and the velocity of their steady-state motion, depending on the velocity of the upward vapor–gas flow in the cooling tower:

$$\Delta T \sim R_0^{-3/2}v_s^{-1/2}.$$

4. It is shown that when $S > 2$ for improving the accuracy of estimates account should be taken of the variation in the parameters of the vapor–air flow (density of water vapor and temperature) in the lower part of the cooling tower as a result of intense evaporation.

NOTATION

x, R, coordinate and radius of droplet; ρ_a and ρ_L, density of air and water; v, velocity of droplet; c, r, specific heat capacity and latent heat of phase transition of water; $\rho(z)$, density of water vapor in the vicinity of a point with the coordinate z; $\rho_s(T)$, density of saturated water vapor at temperature T; u, velocity of vapor–air flow inside the cooling tower; T_a, φ, temperature and relative humidity of the ambient air; D, diffusion factor of water vapor in air; v_s, steady-state velocity of droplet motion, dependening on R; T_l, limiting temperature of transpiration cooling; η, thermal efficiency of the cooling tower; Q and Q_a, specific flow rates of water and air.

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